Int. J. Solids Structures, 1971, Vol. 7, pp. 1007 to 1029. Pergamon Press. Printed in Great Britain

A THEORETICAL STUDY OF THE DYNAMIC PLASTIC BEHAVIOR OF BEAMS AND PLATES WITH FINITE-DEFLECTIONS

NORMAN JONES

Massachusetts Institute of Technology, Department of Naval Architecture and Marine Engineering, Cambridge, Massachusetts 02139

Abstract—An approximate theoretical procedure is developed herein in order to estimate the permanent transverse deflections of beams and arbitrarily shaped plates which are subjected to large dynamic loads. The influence of finite-deflections or geometry changes is retained in the analysis but elastic effects are disregarded. The particular case of a fully clamped rectangular plate acted on by a uniformly distributed dynamic pressure pulse is studied in some detail. It is observed that reasonable agreement between the theoretical predictions and the experimental results has been obtained for beams ($\beta = 0$) and rectangular plates ($\beta = 0.593$) which were made from a strain-rate insensitive material.

NOTATION

B	semi-width of plate
D	internal energy dissipation function per unit length of a plastic hinge
H	thickness of beam or plate
I	$p_0\tau$
I'	$\dfrac{p_0\tau}{(\mu H p_c)^{\frac{1}{2}}}$
L	semi-length of beam or plate
M_{ij}	bending moments per unit length as defined in Fig. 2
M_0	$\dfrac{\sigma_0 H^2}{4}$
N_{ij}	membrane forces per unit length as defined in Fig. 2
N_0	$\sigma_0 H$
Q_i	transverse shear force per unit length as defined in Fig. 2
T	duration of response
V_0	initial velocity
W_f	permanent transverse deflection
W_m	maximum permanent transverse deflection
p_c	magnitude of static collapse pressure
p_i, p_3	external pressures per unit surface area of a plate as defined in Fig. 2
p_0	magnitude of dynamic pressure pulse
t	time
u_i	displacements in mid-plane of a plate as defined in Fig. 1
w	transverse deflection of the mid-plane of a plate or beam as defined in Fig. 1
x_i	coordinates lying in the mid-plane of a plate as defined in Fig. 1
x, x', y	coordinates defined in Fig. 6
z	coordinate which is perpendicular to the mid-plane of an initially flat plate as defined in Fig. 1
β	$\dfrac{B}{L}$
γ	defined by equation (33)
γ_1	defined by equation (B2)

ε_{ij}	direct strain
η	$\frac{p_0}{p_c}$
$\dot{\theta}_m$	relative angular rotation rate across a line hinge
κ_{ij}	curvature
λ	$\frac{\mu V_0^2 L^2}{M_0 H}$
μ	mass per unit area of beam or plate
ξ_0	$\beta \tan \phi$
σ_0	yield stress
τ	duration of a rectangular pressure pulse
ϕ	angle defined in Fig. 6
Γ	defined by equation (35)
$(\dot{\ })$	$\frac{\partial}{\partial t}(\ )$

INTRODUCTION

IN REF. [1], Symonds and Mentel investigated the influence of finite-deflections and axial restraints on the behavior of rigid, perfectly plastic beams loaded impulsively. The authors observed that the permanent deflections of these beams were considerably smaller than those predicted by a simple beam bending analysis when the maximum deflections were of the order of one-half the beam thickness or larger. More recently [2], it has been shown that bending type analyses (infinitesimal theories) of circular plates loaded dynamically only predict reasonable estimates of actual permanent deflections which are less than about one-half the corresponding plate thickness.

It is evident from a literature survey that very few "exact" rigid-plastic solutions which retain the influence of finite-deflections have been published. Moreover, it is observed that changes in the boundary conditions or external loading of the existing solutions often introduce enormous analytical difficulties. In order to circumvent the difficulties encountered in many exact dynamic plastic analyses, Martin presented some bound theorems in Ref. [3]. Martin's theorems may be used to obtain a lower bound to the actual response time and an upper bound to the permanent deflections of a rigid, perfectly plastic continuum subjected to an impulsive loading. However, these theorems are only valid for relatively small permanent deflections of a broad class of structures, since the influence of finite-deflections (geometry changes) have been neglected in their formulation.

A general approximate theoretical procedure, which retains the influence of finite-deflections, is developed herein for the dynamic behavior of arbitrarily shaped, rigid, perfectly plastic plates. This analysis reduces to that presented by Sawczuk [4, 5] for the influence of finite-deflections on the behavior of initially flat rigid, perfectly plastic plates loaded statically, and provides reasonable agreement with the experimental results on beams and rectangular plates reported in Refs. [6, 7].

BASIC RELATIONS

Consider the orthogonal coordinates $x_i (i = 1, 2)$ which lie in the mid-plane of an initially flat plate of thickness H. If the displacement vector of a point on the mid-plane of a plate is chosen with components u_i along the x_i axes and w transverse to the plate, as indicated in Fig. 1, then the equilibrium equations for finite-deflections of the element shown in Fig. 2 can be written,†

$$N_{ij,i}+p_j-\mu\ddot{u}_j = 0 \tag{1a}$$

$$(Q_i+N_{ij}w,_j),_i+p_3-\mu\ddot{w} = 0 \tag{1b}$$

and

$$Q_i = -M_{ji,j} \tag{1c}$$

if rotary inertia is disregarded and the deflections are restricted to moderate values. The positive directions of the internal forces N_{ij}, external loads p_i and p_3, shear forces Q_i and bending moments M_{ij} are defined in Fig. 2.

It may be shown that the associated direct strains

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j}+u_{j,i}+w,_i w,_j) \tag{2a}$$

and curvature changes

$$\kappa_{ij} = w,_{ij} \tag{2b}$$

are consistent with the equilibrium equations (1a)–(1c) according to the principle of virtual velocities.

If attention is restricted henceforth to plates with the following boundary conditions around the outer edges

$$w = \dot{w} = 0 \tag{3a}$$

$$M_{ij}n_i = 0 \tag{3b}$$

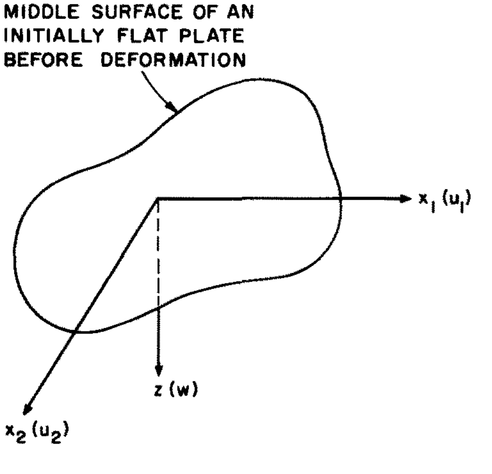

FIG. 1. Middle surface of an initially flat plate before deformation.

† Commas and dots denote differentiation with respect to spatial coordinates x_i and time, respectively. The summation convention is employed throughout this article.

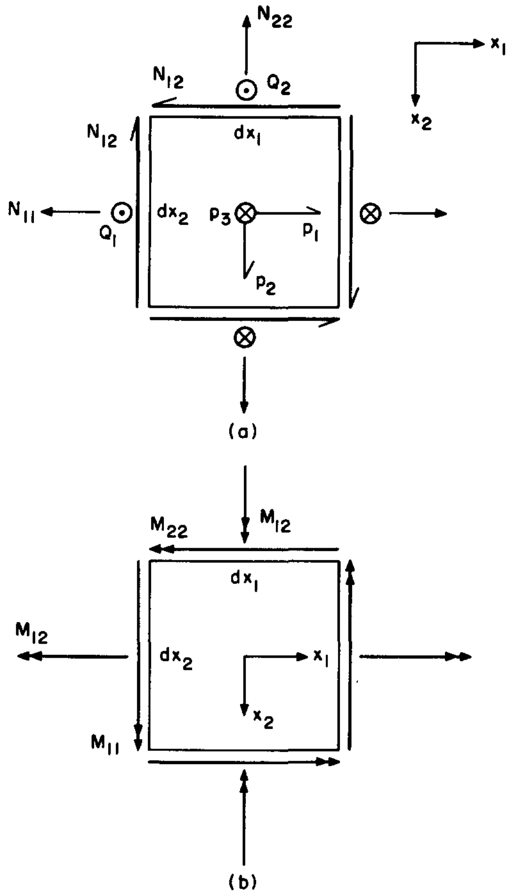

FIG. 2. (a) Forces and (b) moments acting on the middle plane of a plate.

where n_i are the components of a unit normal to the outer boundary, and either

$$N_{ij}n_i = 0 \tag{3c}$$

or

$$u_i = \dot{u}_i = 0 \tag{3d}$$

then the external work rate is

$$D_E = \int_A \{(p_i - \mu\ddot{u}_i)\dot{u}_i + (p_3 - \mu\ddot{w})\dot{w}\}\, \mathrm{d}A. \tag{4}$$

The area A extends over the entire deformed mid-plane of a plate and may be taken as the original area for moderate deflections. Substituting equations (1a) and (1b) into equation (4) gives

$$D_E = -\int_A \{N_{ij,i}\dot{u}_j + (Q_i + N_{ij}w_{,j})_{,i}\dot{w}\}\, \mathrm{d}A. \tag{5}$$

If it is assumed that the total area A of a plate with a boundary curve of length C consists of l smaller areas each of area A_m and surrounded by a boundary curve of length C_m, within which $N_{ij}\dot{u}_j$ is continuous, then, using Green's Theorem, the first term in equation (5) becomes

$$\int_A N_{ij,i}\dot{u}_j \, \mathrm{d}A = \sum_{m=1}^{l} \int_{C_m} N_{ij}\dot{u}_j n_i \, \mathrm{d}C_m - \int_A N_{ij}\dot{u}_{j,i} \, \mathrm{d}A$$

since

$$N_{ij,i}\dot{u}_j = (N_{ij}\dot{u}_j)_{,i} - N_{ij}\dot{u}_{j,i}$$

A somewhat similar procedure can be applied to the remaining terms in equation (5). It may be shown that these additional operations allow equation (4) to be recast in the form

$$\begin{aligned} \int_A \{(p_i - \mu\ddot{u}_i)\dot{u}_i + (p_3 - \mu\ddot{w})\dot{w}\} \, \mathrm{d}A = &- \sum_{m=1}^{l} \int_{C_m} N_{ij}\dot{u}_j n_i \, \mathrm{d}C_m \\ &+ \int_A N_{ij}\dot{u}_{j,i} \, \mathrm{d}A + \sum_{m=1}^{q} \int_{C_m} M_{ji,j}\dot{w} n_i \, \mathrm{d}C_m - \sum_{m=1}^{r} \int_{C_m} M_{ji}\dot{w}_{,i} n_j \, \mathrm{d}C_m \\ &+ \int_A M_{ji}\dot{w}_{,ji} \, \mathrm{d}A - \sum_{m=1}^{s} \int_{C_m} N_{ij} w_{,j}\dot{w} n_i \, \mathrm{d}C_m + \sum_{m=1}^{t} \int_{C_m} N_{ij} w\dot{w}_{,i} n_j \, \mathrm{d}C_m \\ &- \int_A (N_{ij,j} w\dot{w}_{,i} + N_{ij} w\dot{w}_{,ji}) \, \mathrm{d}A. \end{aligned} \tag{6}$$

Clearly the transverse deflection w and the transverse velocity $\dot{w}$ must be continuous throughout an entire plate made from a homogeneous material. In plates made from a perfectly plastic material with a yield criterion $\phi(N_{ij}, M_{ij}) = \text{const.}$, $w_{,i}$ is continuous except possibly at any time-independent locations where $N_{ij} = 0$. The time derivative of the slope ($\dot{w}_{,i}$) can be discontinuous at travelling plastic hinges in plates made from a perfectly plastic material. Moreover, Q_i, M_{ij} and N_{ij} must be continuous throughout an entire plate except possibly when Q_i, M_{ij} $(i = j)$ and N_{ij} at a plastic hinge act on a plane perpendicular to the hinge line.

The above observations and equations (3a)–(3d) allow equation (6) to be rewritten

$$\begin{aligned} \int_A \{(p_i - \mu\ddot{u}_i)\dot{u}_i + (p_3 - \mu\ddot{w})\dot{w}\} \, \mathrm{d}A = &- \sum_{m=1}^{l} \int_{C_m} N_{ij}\dot{u}_j n_i \, \mathrm{d}C_m \\ &+ \int_A N_{ij}\dot{u}_{j,i} \, \mathrm{d}A + \sum_{m=1}^{r} \int_{C_m} (N_{ij} w - M_{ji})\dot{w}_{,i} n_j \, \mathrm{d}C_m \\ &- \int_A N_{ij,j} w\dot{w}_{,i} \, \mathrm{d}A + \int_A (M_{ji} - N_{ij} w)\dot{w}_{,ji} \, \mathrm{d}A. \end{aligned} \tag{7}$$

It may be shown that equation (7) reduces to equation (16) in Ref. [4] which was developed by Sawczuk for the finite-deflections of flat plates loaded statically. This solution may be further specialized to give a kinematically admissible collapse load for infinitesimal deflections.

APPROXIMATE ANALYSIS

The forces, moments and deflections appearing in equation (7) are interconnected by a function $\Phi(N_{ij}, M_{ij}, u_i, w) = \text{const.}$ which controls the yielding of a perfectly plastic material. Thus, if the finite-deflection terms are retained in equation (7) then it is difficult to obtain the load–deflection behavior for static loads or the deflection–time history for dynamic loads. In order to simplify (7) it might be noted that it is usually reasonable to neglect the influence of u_i and $\ddot{u}_i$ compared to w and $\ddot{w}$ when $p_i = 0$. Thus, using equation (1a), equation (7) in this case becomes

$$\int (p_3 - \mu\ddot{w})\dot{w}\,\mathrm{d}A = \sum_{m=1}^{r} \int_{C_m} (N_{ij}w - M_{ji})\dot{w}_{,i}n_j\,\mathrm{d}C_m + \int_A (M_{ji} - N_{ij}w)\dot{w}_{,ji}\,\mathrm{d}A. \tag{8}$$

The first term on the right hand side of (8) gives the internal energy dissipated at any travelling plastic "hinges" while the remaining term in (8) is the energy dissipated in continuous deformation fields. If the deformation field has no discontinuities in $\dot{w}_{,i}$ across the curves C_m then, in order to satisfy equations (3a) and (3b), the first term in (8) vanishes. Nevertheless, even with these simplifications, equation (8) is difficult to solve except for particular cases in which the directions of principal stresses are known, i.e. beams and axisymmetrically loaded circular plates. However, some members of this class of structures have been studied using other methods of solution [1, 2, etc.]. It is of principal interest in this article to develop an approximate procedure which may be used in order to predict the dynamic behavior of non-axisymmetric plates for which no solutions exist except the special case of a simply supported square plate [8].

Sawczuk [4, 5] examined the static behavior of a flat plate by dividing it into a number of rigid regions which were separated by time-independent hinges situated at locations where discontinuities in $w_{,i}$ and $\dot{w}_{,i}$ occurred. This procedure is clearly only approximate but it permitted Sawczuk to analyze the post yield behavior of rectangular plates loaded statically. Sawczuk and Winnicki [9] conducted some experiments on rectangular reinforced concrete plates and obtained rather good agreement with the predictions of the theoretical procedure developed in Refs. [4, 5]. It is evident from infinitesimal plasticity analyses that the shape of the displacement field of a beam or plate for small dynamic loads is the same as the static collapse velocity profile [8, etc.]. Moreover, the experimental results reported in Ref. [7] indicate that the permanent deformed profiles of rectangular plates loaded dynamically are similar to the shape of the velocity field used by Wood [10] in order to calculate the minimum upper bound to the collapse pressure of the corresponding static problem. Thus, in view of these observations and in order to simplify the study of non-axisymmetric plates, the approximations introduced by Sawczuk [4] for the static behavior of plates will now be incorporated into equation (8) to give

$$\int_A (p_3 - \mu\ddot{w})\dot{w}\,\mathrm{d}A = \sum_{m=1}^{r} \int_{C_m} (N_{ij}w - M_{ji})\dot{w}_{,i}n_j\,\mathrm{d}C_m \tag{9}$$

where C_m are time-independent. Equation (9) reduces to the solution obtained by Sawczuk [4] for static finite-deflections and gives a kinematically admissible solution for the infinitesimal behavior of a plate or beam loaded dynamically.

If a flat plate is divided into a number of rigid regions separated by p straight line hinges, each of length l_m, then equation (9) further simplifies to

$$\int_A (p_3 - \mu\ddot{w})\dot{w}\,\mathrm{d}A = \sum_{m=1}^{p} \int_{l_m} (Nw - M)\dot{\theta}_m\,\mathrm{d}l_m \tag{10}$$

where w, on the right hand side of equation (10), is the transverse deflection at the hinge. The stresses at the hinge due to the axial force N and bending moment M act on a plane which is parallel to the hinge and transverse to the mid-plane of the plate. $\dot{\theta}_m$ is the relative angular rotation rate across the hinge.

It is convenient to define

$$D = (Nw - M)\dot{\theta}_m \tag{11}$$

which is the internal energy dissipation per unit length of a hinge. Clearly, the dissipation function D will depend on the type of supports around the boundary of a plate and on the yield condition which is selected. If the maximum normal stress yield criterion is chosen, then, in order to satisfy the normality requirements of plasticity and the Euler–Bernoulli assumption we have

$$D = M_0\left(1 + 4\frac{w^2}{H^2}\right)\dot{\theta}_m \quad \text{when} \quad \frac{w}{H} \leq \tfrac{1}{2} \tag{12a}$$

and

$$D = 4M_0\frac{w}{H}\dot{\theta}_m \quad \text{when} \quad \frac{w}{H} \geq \tfrac{1}{2} \tag{12b}$$

for a hinge in a beam or plate with simply supported edges. On the other hand, if a hinge is located in the interior of a beam or plate with clamped edges, then

$$D = M_0\left(1 + \frac{3w^2}{H^2}\right)\dot{\theta}_m \quad \text{when} \quad \frac{w}{H} \leq 1 \tag{13}$$

while equation (12b) now holds, provided $w/H \geq 1$. Equations (12) and (13) remain valid for hinges which are inclined at any angle to the supports of a plate.

It should be remarked that equation (10) was derived for an arbitrarily shaped plate having boundary conditions which satisfy equations (3a)–(3d). However, since equation (10) may be interpreted as an energy balance it can be used for plates which have other edge conditions. In this case it might be necessary to allow plastic hinges to form around the boundary of the plate as well as in the interior.

If a kinematically admissible collapse mechanism with straight line hinges is postulated, then equation (10), combined with the appropriate relation for D, may be solved to give an estimate of the influence of finite-deflections on the deflection–time history of a non-axisymmetric rigid, perfectly plastic plate loaded dynamically. As noted in the previous discussion it is believed that, for moderate values of permanent deflections, the static collapse velocity profile would be a reasonable choice for the displacement field in the corresponding dynamic problem. Clearly, because no uniqueness or bounding theorems have been proved herein, it is impossible to know whether the predictions of equation (10) using a particular displacement field are smaller or larger than the exact solution. However, this uncertainty is characteristic of most studies into the influence of finite-deflections on the behavior of structures loaded either statically or dynamically. Nevertheless, the results predicted by equation (10) for the cases discussed in the next section are considerably closer to the corresponding experimental values than are the estimates obtained using infinitesimal theories and should be fairly reliable in other cases, provided physically reasonable displacement fields are selected. It is believed that these predictions should be sufficiently accurate to enable designers to make preliminary design decisions. The numerical procedures of Ref. [11, etc.] could then be used in order to obtain further details of the final design which may be required.

DYNAMIC BEHAVIOR OF A BEAM

Consider the simply supported rigid, perfectly plastic beam of length $2L$ and thickness H which is indicated in Fig. 3. The beam is subjected to a uniformly distributed dynamic load which has the pressure–time history shown in Fig. 4. This external loading may be expressed in the form

$$p_3 = p_0 \quad \text{for } 0 \le t \le \tau \tag{14a}$$

and

$$p_3 = 0 \quad \text{for } \tau \le t \le T \tag{14b}$$

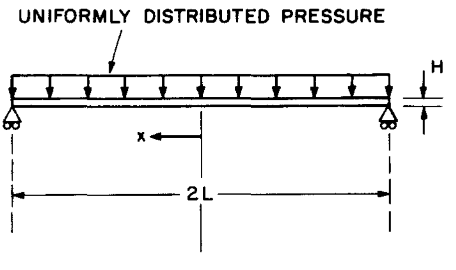

FIG. 3. Simply supported beam.

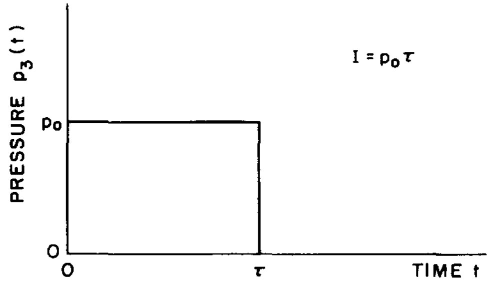

FIG. 4. Dynamic pressure pulse.

where τ and T are the durations of the pressure pulse and the beam response, respectively. If it is assumed that the shape of the displacement field due to dynamic loads which produce finite-deflections is the same as the velocity profile developed for the corresponding static collapse load, then

$$w = w_1\left(1-\frac{x}{L}\right) \quad \text{when } 0 \leq t \leq \tau \tag{15a}$$

and

$$w = w_2\left(1-\frac{x}{L}\right) \quad \text{when } \tau \leq t \leq T \tag{15b}$$

provided $0 \leq x \leq L$.

Now, substituting equations (12a), (14a) and (15a) into (10) gives

$$\ddot{w}_1 + hw_1^2 = \mathrm{d} \tag{16}$$

where

$$0 \leq t \leq \tau$$

$$\frac{w_1}{H} \leq \frac{1}{2}$$

$$h = \frac{6p_c}{\mu H^2} \tag{17a}$$

$$d = \frac{3p_c}{2\mu}(\eta - 1) \tag{17b}$$

$$p_c = \frac{2M_0}{L^2} \tag{17c}$$

$$\eta = \frac{p_0}{p_c} \tag{17d}$$

and

$$(\dot{\ }) = \frac{\mathrm{d}}{\mathrm{d}t}(\ ).$$

A solution to equation (16) may be obtained using the method of successive approximations. Thus, when using a second approximation to the solution of equation (16) which satisfies the initial conditions $w = \dot{w} = 0$ at $t = 0$, it may be shown that

$$w_1 = \bar{w}_1$$

and

$$\dot{w}_1 = \dot{\bar{w}}_1$$

at the end of the first stage of motion ($t = \tau$), where

$$\bar{w}_1 = \frac{\mathrm{d}\tau^2}{2}\left(1 - \frac{h\,\mathrm{d}\tau^4}{60}\right) \tag{18a}$$

and

$$\dot{w}_1 = d\tau\left(1-\frac{h\,d\tau^4}{20}\right). \tag{18b}$$

A study of the second stage of motion ($\tau \le t \le T$) proceeds in a manner similar to that outlined above for the first stage, but with the initial conditions $w = \dot{w} = 0$ replaced by (18a) and (18b), and with $\eta = 0$. It may be shown that the maximum value of the transverse deflections of the permanently deformed beam is

$$\frac{W_m}{H} = \frac{\beta_5}{\beta_6}+\beta_7\rho+\frac{(\beta_1-\beta_5^2)\rho^2}{2\beta_6}-\frac{\beta_5\beta_7\rho^3}{3}-\left(\frac{\beta_1\beta_5}{\beta_6}+\beta_6\beta_7^2\right)\frac{\rho^4}{12}-\frac{\beta_1\beta_7\rho^5}{20}-\frac{\beta_1^2\rho^6}{120\beta_6} \tag{19}$$

where

$$\rho = \frac{T-\tau}{\tau}$$

is the smallest root $\rho \ge 0$ of the polynomial equation

$$-\frac{\beta_1\rho^5}{20}-\frac{\beta_1\beta_2\rho^4}{4}-\frac{(\beta_5+\beta_1\beta_2^2)\rho^3}{3}-\beta_2\beta_5\rho^2+(1-\beta_4)\rho+\beta_2 = 0. \tag{20}$$

Equation (20) is obtained from the requirement that $\dot{w} = 0$ at $t = T$, and

$$\beta_1 = -\frac{9I'^4}{\eta^4} \tag{21a}$$

$$\beta_2 = (1-\eta)(1-\beta_3) \tag{21b}$$

$$\beta_3 = \frac{9(\eta-1)I'^4}{20\eta^4} \tag{21c}$$

$$\beta_4 = -\frac{9I'^4}{4\eta^4}(\eta-1)^2\left(1-\frac{\beta_3}{3}\right)^2 \tag{21d}$$

$$\beta_5 = \frac{9I'^4}{2\eta^4}(\eta-1)\left(1-\frac{\beta_3}{3}\right) \tag{21e}$$

$$\beta_6 = \frac{6I'^2}{\eta^2} \tag{21f}$$

$$\beta_7 = \frac{\beta_1\beta_2}{\beta_6} \tag{21g}$$

and

$$I' = \frac{p_0\tau}{(\mu H p_c)^{\frac{1}{2}}} \tag{21h}$$

As remarked previously, equation (19) remains valid provided $W_m/H \le \frac{1}{2}$. If it is assumed that w/H exceeds one-half when $t \ge \tau$, then the first stage of motion given by equations (16)–(18b) remains unchanged, while the second stage remains valid until $t = t_2$, when $w/H = \frac{1}{2}$. Thus, the non-dimensionalised time $\rho_2 = (t_2-\tau)/\tau$ may be found from equation (19) with $W_m/H = \frac{1}{2}$, while the corresponding transverse velocity of the beam is

$$\dot{w} = \dot{W}_2\left(1-\frac{x}{L}\right)$$

where $\dot{W}_2$ is obtained by observing that the left hand side of equation (20) with $\rho = \rho_2$ equals $\dot{W}_2/(\bar{d}\tau)$. Now all subsequent behavior of the beam will have $w/H \geqq \frac{1}{2}$ at the centrally located plastic hinge. Thus, it is necessary to consider a third stage of motion which is governed by the dissipation relation (12b). It is straightforward to show that the permanent shape of the beam is

$$\frac{W_f}{H} = \frac{1}{2}\left\{1+\frac{\beta_6}{4}\left(\frac{\dot{W}_2}{\bar{d}\tau}\right)^2\right\}^{\frac{1}{2}}\left(1-\frac{x}{L}\right) \tag{22}$$

provided $0 \leq x \leq L$, $\dot{W}_2 \geq 0$ and $\bar{d} = -3p_c/2\mu$.

The foregoing analysis is simplified considerably when employing the square yield condition shown in Fig. 5, for which the corresponding dissipation relation is

$$D = M_0\left(1+\frac{4w}{H}\right)\dot{\theta}_m. \tag{23}$$

It may be shown, when using (23), that the permanent shape of a simply supported beam loaded with a uniformly distributed rectangular pressure pulse is

$$\frac{W_f}{H} = \frac{1}{4}[\{2\eta(\eta-1)(1-\cos(hH\tau^2)^{\frac{1}{2}})+1\}^{\frac{1}{2}}-1]\left(1-\frac{x}{L}\right). \tag{24}$$

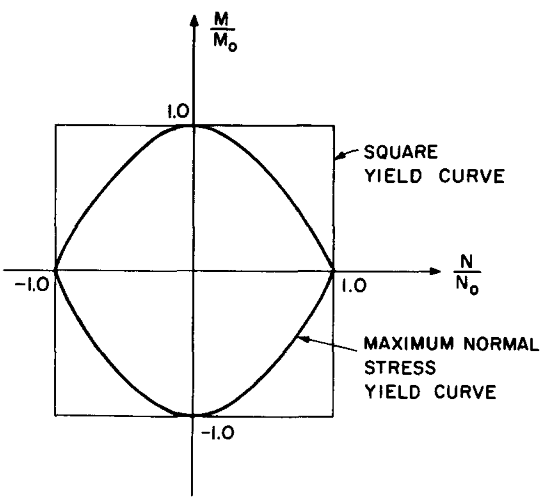

FIG. 5. Yield curves.

Equation (24) reduces to

$$\frac{W_f}{H} = \frac{1}{4}[(1+3\lambda)^{\frac{1}{2}}-1]\left(1-\frac{x}{L}\right) \tag{25a}$$

for impulsive loadings when $p_0\tau = \mu V_0$ and

$$\lambda = \frac{\mu V_0^2 L^2}{M_0 H}. \tag{25b}$$

A beam with fully clamped ends and subjected to a uniformly distributed rectangular pressure pulse may be analyzed in the same manner as that described above for the simply supported case. The corresponding predictions for the clamped cases are presented in Appendices A and B.

DYNAMIC BEHAVIOR OF A RECTANGULAR PLATE

Consider a rigid, perfectly plastic rectangular plate of length $2L$ and width $2B$ which is fully clamped around the outer boundary as indicated in Fig. 6. The plate is subjected to a uniformly distributed dynamic load with the pressure–time history shown in Fig. 4 and described by equations (14a) and (14b). It is assumed that the shape of the displacement field for the dynamic case is the same as the velocity profile used by Wood [10] to give an upper bound to the collapse load of the corresponding static problem. Thus,

$$w = \frac{W_i(B\tan\phi - x')}{B\tan\phi} \tag{26a}$$

for region I
and

$$w = \frac{W_i(B-y)}{B} \tag{26b}$$

for region II. Regions I and II are indicated in Fig. 6 and $i = 1$ refers to the deflections during the time interval $0 \le t \le \tau$, while $i = 2$ corresponds to $\tau \le t \le T$.

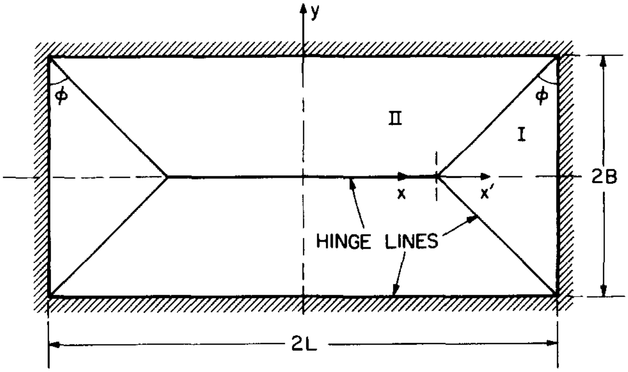

FIG. 6. Plastic hinge line pattern in a fully clamped rectangular plate subjected to a uniformly distributed transverse pressure.

If equations (13), (14a), (26a) and (26b) are substituted into equation (10) then it may be shown that

$$\ddot{W}_1 + h_1 W_1^2 = d_1 \tag{27}$$

where

$$h_1 = \frac{p_c(4\xi_0^2 - 11\xi_0 + 9)}{3\mu H^2(2-\xi_0)} \tag{28a}$$

$$d_1 = \frac{p_c(\eta - 1)(3-\xi_0)}{\mu(2-\xi_0)} \tag{28b}$$

$$p_c = \frac{12M_0}{B^2(3-2\xi_0)} \tag{28c}$$

$$\eta = \frac{p_0}{p_c} \tag{28d}$$

$$\xi_0 = \beta \tan \phi \tag{28e}$$

and

$$\beta = \frac{B}{L} \tag{28f}$$

provided $W_1/H \leq 1$ and $0 \leq t \leq \tau$. If equation (27) is solved using the method of successive approximations then a second approximation to the exact solution which satisfies the initial conditions $w = \dot{w} = 0$ at $t = 0$, gives equations (18a) and (18b) at $t = \tau$ provided h and d are replaced by h_1 and d_1, respectively.

The second stage of motion ($\tau \leq t \leq T$) may be studied in a manner similar to the method outlined for the first stage but with $\eta = 0$ and the initial conditions replaced by the appropriate equations which correspond to (18a) and (18b). It may be shown that the maximum value of the transverse deflections of the permanently deformed plate and the response time are given by equations (19) and (20), respectively. In the present case the constants β_i, where $i = 1, 2, \ldots, 7$, must now be evaluated by replacing the beam coefficients (17a)–(17d) by the plate coefficients (28a)–(28d), respectively.

It was remarked previously that the analysis outlined above remains valid provided $W_m/H \leq 1$. If this inequality is violated then the internal energy dissipation is governed by the dissipation relation (12b) in those portions of the hinge lines which have transverse deflections greater than the plate thickness. Thus, a time-dependent rectangular shaped boundary travels outwards from the central line hinge towards the edge of a plate. This boundary always has a deflection $w = H$ and divides the plate into two regions: an outer zone which has $w < H$ and an inner zone with $w > H$. The internal energy dissipation in the outer region is controlled by equation (13), while equation (12b) governs the behavior in the remainder of the plate. If the response of the plate is divided into three stages as described for the beam studied previously, then it may be shown that the maximum permanent deflection W_m is given by the root of the transcendental equation

$$\frac{\beta_8}{2}\left(\frac{W_m}{H}\right)^2 + 2\beta_9 \log_e\left(\frac{W_m}{H}\right) = \frac{\beta_8}{2} + \left(\frac{\dot{W}_2 \tau}{H}\right)^2 \tag{29}$$

where

$$\beta_8 = \frac{4I'^2}{\eta^2}\left\{1+\frac{(1-\xi_0)^2}{2-\xi_0}\right\} \tag{30a}$$

and

$$\beta_9 = \frac{2I'^2\xi_0}{3\eta^2}. \tag{30b}$$

The non-dimensionalised velocity of the plate centre at the end of the second stage of motion $\dot{W}_2\tau/H$ equals the left hand side of equation (20) multiplied by β_1/β_6, while the non-dimensionalised duration of the second stage [i.e. $\rho = (t_2-\tau)/\tau$] is obtained from equation (19) with $W_m/H = 1$. Clearly the values of β_i ($i = 1, 2, \ldots, 7$) in these equations are evaluated using (28a)–(28d), which correspond to the rectangular plate.

The dissipation relation (23) derived for a square yield curve simplifies considerably the analysis for the dynamic behavior of a fully clamped rectangular plate with finite-deflections. It may be shown that the duration of response of the plate is

$$T = \frac{1}{\gamma}\left[\tan^{-1}\left\{\frac{\eta\sin(\gamma\tau)}{\eta\cos(\gamma\tau)+1-\eta}\right\}\right] \tag{31}$$

and that the maximum permanent deflection is

$$\frac{W_m}{H} = (3-\xi_0)\frac{\{[1+2\eta(\eta-1)\{1-\cos(\gamma\tau)\}]^{\frac{1}{2}}-1\}}{2\{1+(2-\xi_0)(1-\xi_0)\}} \tag{32}$$

where

$$\gamma^2 = \frac{24M_0}{\mu HB^2(3-2\xi_0)}\left\{1-\xi_0+\frac{1}{2-\xi_0}\right\}. \tag{33}$$

Equation (32) gives

$$\frac{W_m}{H} = \frac{(3-\xi_0)\{(1+\Gamma)^{\frac{1}{2}}-1\}}{2\{1+(\xi_0-1)(\xi_0-2)\}} \tag{34}$$

for impulsive loadings ($I = p_0\tau = \mu V_0$) where λ, ξ_0 and β are defined by equations (25b), (28e) and (28f), respectively, and

$$\Gamma = \frac{\lambda\beta^2}{6}(3-2\xi_0)\left(1-\xi_0+\frac{1}{2-\xi_0}\right). \tag{35}$$

A simply supported rectangular plate which is subjected to a uniformly distributed rectangular pressure pulse may be analysed in the same manner as outlined above for the fully clamped case. The corresponding predictions for a simply supported rectangular plate are presented in Appendices A and B.

DISCUSSION

The approximate procedure outlined in this article has been used to predict the permanent transverse deflections (W_m/H) presented in Figs. 7–11 for fully clamped rectangular plates which are subjected to uniformly distributed dynamic pressure pulses. It is evident

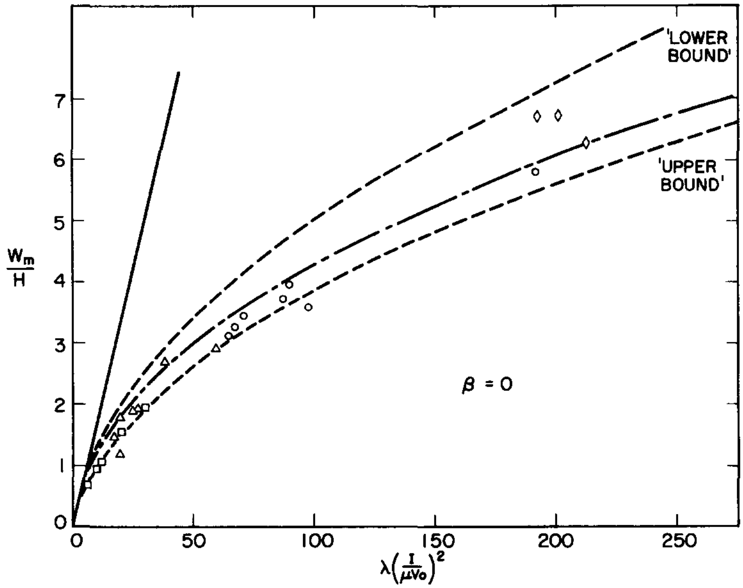

FIG. 7. Maximum permanent transverse deflection vs. a non-dimensionalised impulse parameter for a fully clamped beam ($\beta = 0$) which is subjected to a uniformly distributed dynamic pressure: ——— bending only theory for an impulsive pressure; – – – – finite-deflection analysis for an impulsive pressure using a square yield curve; — — — finite-deflection analysis for a dynamic pressure ($\eta = 100$) using a maximum normal stress yield curve. Experimental results on wide Al 6061-T6 beams which are subjected to a uniformly distributed impulsive pressure ($\eta \to \infty$) [6]: ◇ $2L/H = 56{\cdot}7$; o $2L/H = 41{\cdot}35$; △ $2L/H = 26{\cdot}7$; □ $2L/H = 20{\cdot}5$.

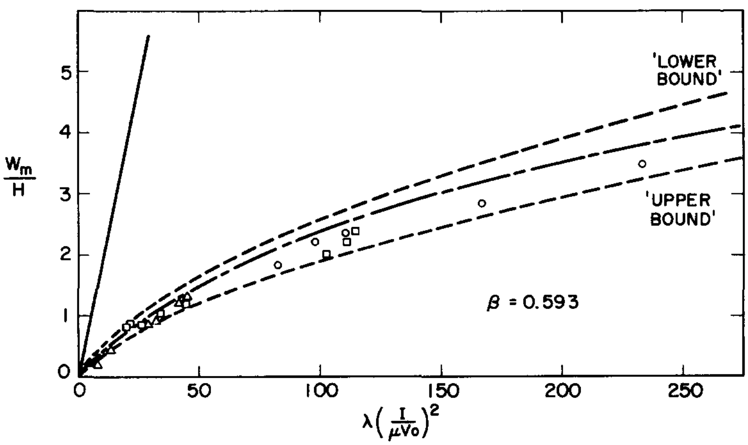

FIG. 8. Maximum permanent transverse deflection vs. a non-dimensionalised impulse parameter for a fully clamped rectangular plate ($\beta = 0{\cdot}593$) which is subjected to a uniformly distributed dynamic pressure; ——— upper bound to bending only theory for an impulsive pressure, equation (10) of [7]; – – – – finite-deflection analysis for an impulsive pressure using a square yield curve; — — — finite-deflection analysis for a dynamic pressure ($\eta = 100$) using a maximum normal stress yield curve. Experimental results on rectangular plates ($2L = 5{\cdot}0625$ in., $2B = 3$ in., $\beta = 0{\cdot}593$) made from Al 6061-T6 and subjected to a uniformly distributed impulsive pressure ($\eta \to \infty$) [7]: o $H = 0{\cdot}122$ in.; □ $H = 0{\cdot}188$ in.; △ $H = 0{\cdot}244$ in.

from Figs. 7 and 8 that the estimates which were obtained using the maximum normal stress yield criterion for beams ($\beta = 0$) and rectangular plates with $\beta = 0{\cdot}593$ agree reasonably well with the corresponding experimental values. It should be noted that the experimental results plotted in Fig. 7 were recorded on wide beams. However, it is shown in Ref. [6] that the permanent deflections of wide beams or rectangular plates which are clamped on two opposite edges and free on the remaining two sides are almost independent of the aspect ratio. The test specimens used in the experiments reported in Refs. [6, 7] and presented in Figs. 7 and 8 were made from Al 6061-T6 which may be regarded as insensitive to strain-rate up to 10^3 in./in./sec [12] and were subjected to uniformly distributed impulsive pressures ($\eta \to \infty$).

It may be shown that the square yield curve indicated in Fig. 5 circumscribes the corresponding maximum normal stress yield curve while another with dimensions 0·618 times as large would inscribe it. Thus, the "upper bound" results in Figs. 7–10 were predicted by a theoretical solution using a square yield curve which circumscribes the maximum normal stress yield curve, while the "lower bound" values shown in Figs. 7 and 8 correspond to an inscribing yield curve.

It is evident from Figs. 9 and 10 that, for a given value of impulse, the permanent transverse deflection of a beam or plate is essentially independent of the magnitude of η when η is larger than 100.

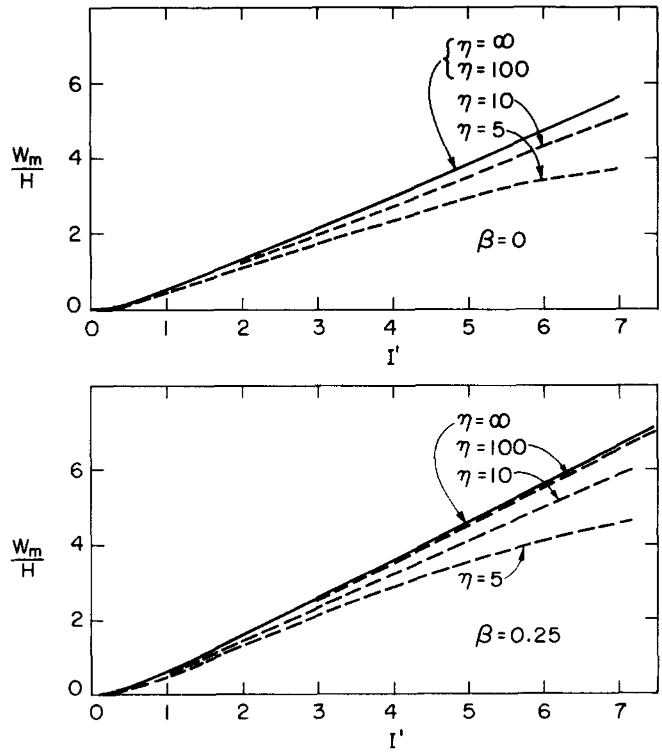

FIG. 9. W_m/H vs. I' predicted by a square yield curve solution for a fully clamped beam ($\beta = 0$) and a fully clamped rectangular plate ($\beta = 0{\cdot}25$) which are subjected to uniformly distributed dynamic pressures.

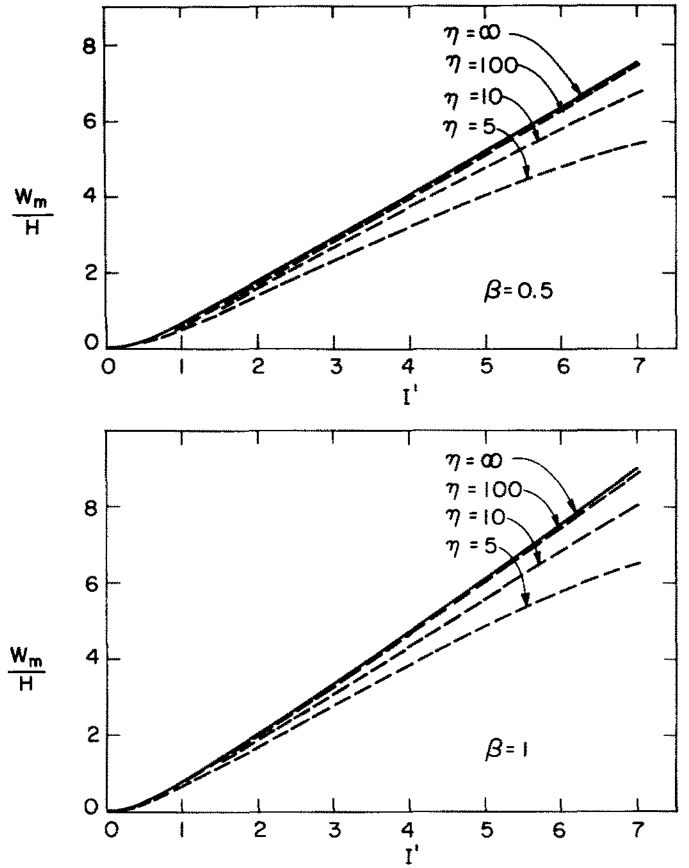

FIG. 10. W_m/H vs. I' predicted by a square yield curve solution for fully clamped rectangular plates with $\beta = 0{\cdot}5$ and $\beta = 1$ which are subjected to uniformly distributed dynamic pressures.

All the theoretical results presented in this article were obtained by assuming that angular changes across the plastic hinges were sufficiently small so that $\tan\theta$ could be replaced by θ rad. This simplification provides a good approximation when the L/H ratio of a beam is large or the value of W_m/H is not too large. Moreover, only the second approximation was retained in the series solution of the differential equation which was encountered in the analysis employing the maximum normal stress yield criterion. It would be necessary to consider additional terms for large W_m/H ratios and for cases in which the dynamic pressure to static pressure ratio (η) is small.

The theoretical work presented herein utilizes time-independent displacement profiles which have a shape similar to the corresponding static collapse fields. Keil [13] has indicated, however, that the deformed shapes of circular plates loaded dynamically are different to those loaded statically. Furthermore, the results presented herein only apply to plates made from strain-rate insensitive materials. Various authors have shown that it is necessary to retain this affect for strain-rate sensitive materials such as mild steel [14, etc.].

The approximate procedure discussed in this article could be used to examine the influence of large transverse dynamic loads on the behavior of beams and plates which have any shape and support conditions. It would be necessary when studying the dynamic

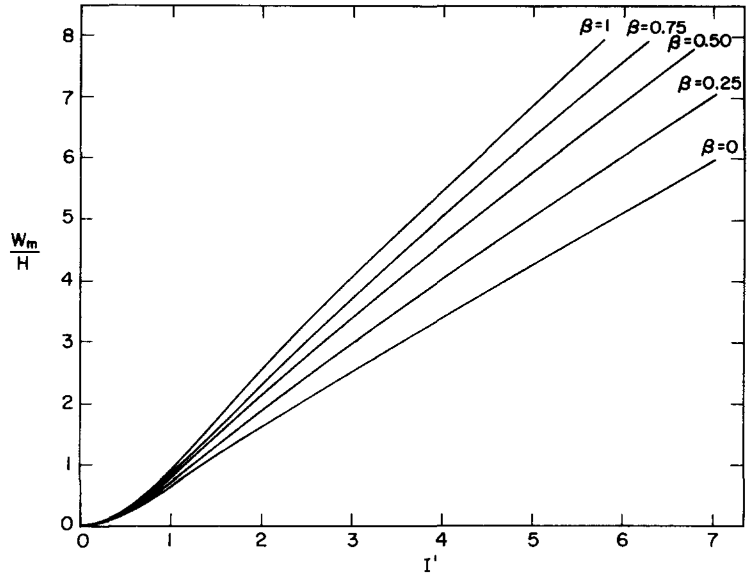

FIG. 11. Variation of W_m/H vs. I' predicted by a maximum normal stress yield curve solution for fully clamped rectangular plates with various values of β which are subjected to uniformly distributed dynamic pressures ($\eta = 100$).

behavior of certain plates to permit continuous deformation fields. This may be accomplished by retaining the last integral on the right hand side of equation (8).

Finally, it should be remarked that the predictions of rigid, perfectly plastic analyses are thought to be useful when the external dynamic energy applied to a structure is about ten times larger than the amount of energy which could be absorbed by the structure in a wholly elastic manner [15].

CONCLUSIONS

An approximate theoretical procedure is developed herein in order to estimate the permanent transverse deflections of beams and arbitrarily shaped plates which are subjected to large dynamic loads. The influence of finite-deflections or geometry changes is retained in the analysis but elastic effects are disregarded. The particular case of a fully clamped rectangular plate acted on by a uniformly distributed dynamic pressure pulse is studied in some detail. It is observed that reasonable agreement between the theoretical predictions and the experimental results has been obtained for beams ($\beta = 0$) and rectangular plates ($\beta = 0{\cdot}593$) which were made from a strain-rate insensitive material.

Acknowledgments—The author is indebted to O.N.R. for their support of this work under Contract No. N00014-67-A-0204-0032. The author wishes to take this opportunity to record his appreciation to Mr. C. Rengos for assistance with the numerical calculations.

REFERENCES

[1] P. S. SYMONDS and T. J. MENTEL, Impulsive loading of plastic beams with axial restraints. *J. Mech. Phys. Solids* **6**, 186–202 (1958).
[2] N. JONES, Finite deflections of a rigid-viscoplastic strain-hardening annular plate loaded impulsively. *J. appl. Mech.* **35**, 349–356 (1968).
[3] J. B. MARTIN, Impulsive loading theorems for rigid-plastic continua. *Proc. Am. Soc. civ. Engrs* **90**, 27–42 (1964).
[4] A. SAWCZUK, On initiation of the membrane action in rigid-plastic plates. *J. Méc.* **3**, 15–23 (1964).
[5] A. SAWCZUK, Large deflections of rigid-plastic plates, *Proc. 11th Int. Cong. Appl. Mech.*, pp. 224–228 (1964).
[6] N. JONES, R. N. GRIFFIN and R. E. VAN DUZER, An experimental study into the dynamic plastic behavior of wide beams and rectangular plates, *Int. J. Mech. Sci.*, to be published.
[7] N. JONES, T. O. URAN and S. A. TEKIN, The dynamic plastic behavior of fully clamped rectangular plates. *Int. J. Solids Struct.* **6**, 1499–1512 (1970).
[8] A. D. COX and L. W. MORLAND, Dynamic plastic deformations of simply-supported square plates. *J. Mech. Phys. Solids* **7**, 229–241 (1959).
[9] A. SAWCZUK and L. WINNICKI, Plastic behavior of simply supported reinforced concrete plates at moderately large deflections. *Int. J. Solids Struct.* **1**, 97–111 (1965).
[10] R. H. WOOD, *Plastic and Elastic Design of Slabs and Plates*. Ronald Press (1961).
[11] J. W. LEECH, E. A. WITMER and T. H. H. PIAN, A numerical calculation technique for large elasto-plastic transient deformations of thin shells. *AIAA Jnl.* **6**, 2352–2359 (1968).
[12] C. J. MAIDEN and S. J. GREEN, Compressive strain-rate tests on six selected materials at strain rates from 10^{-3} to 10^4 in./in./sec. *J. appl. Mech.* **33**, 496–504 (1966).
[13] A. H. KEIL, Problems of plasticity in naval structures: explosive and impact loading, *Proc. Second Symp. on Naval Structural Mech.*, edited by E. H. LEE and P. S. SYMONDS. Pergamon Press (1961).
[14] P. S. SYMONDS, Viscoplastic behavior in response of structures to dynamic loading, *Behavior of Materials Under Dynamic Loading*, edited by N. J. HUFFINGTON, pp. 106–124. A.S.M.E. (1965).
[15] P. S. SYMONDS, Survey of methods of analysis for plastic deformation of structures under dynamic loading, Brown University Report BU/NSRDC/1-67 (1967).

APPENDIX A

The procedure outlined in the main text may be used to study the influence of uniformly distributed pressure pulses on the dynamic behavior of beams and rectangular plates which are either simply supported or fully clamped at the supports. It will be shown in this section how the results for these particular cases may be expressed in a common form when the beams and plates are made from a rigid, perfectly plastic material which yields according to maximum normal stress yield condition.

(a) *Simply supported beams and plates with $W_m/H \le \frac{1}{2}$ and fully clamped beams and plates with $W_m/H \le 1$*

It may be shown that the maximum permanent transverse deflection for all cases is

$$\frac{W_m}{H} = \frac{\beta_5}{\beta_6} + \beta_7\rho + \frac{(\beta_1 - \beta_5^2)\rho^2}{2\beta_6} - \frac{\beta_5\beta_7\rho^3}{3} - \left(\frac{\beta_1\beta_5}{\beta_6} + \beta_6\beta_7^2\right)\frac{\rho^4}{12} - \frac{\beta_1\beta_7\rho^5}{20} - \frac{\beta_1^2\rho^6}{120\beta_6} \quad \text{(A1)}$$

where the non-dimensionalised duration of the second stage (ρ) is the smallest root for which $\rho \ge 0$ of the polynomial equation

$$\beta_2 + (1-\beta_4)\rho - \beta_2\beta_5\rho^2 - \frac{(\beta_5 + \beta_1\beta_2^2)\rho^3}{3} - \frac{\beta_1\beta_2\rho^4}{4} - \frac{\beta_1\rho^5}{20} = 0 \quad \text{(A2)}$$

where

$$\beta_1 = h\bar{d}\tau^4 \tag{A3a}$$

$$\beta_2 = \frac{\dot{W}_1}{\bar{d}\tau} \tag{A3b}$$

$$\beta_3 = \frac{hd\tau^4}{20} \tag{A3c}$$

$$\beta_4 = \frac{h\overline{W}_1^2}{\bar{d}} \tag{A3d}$$

$$\beta_5 = h\overline{W}_1\tau^2 \tag{A3e}$$

$$\beta_6 = hH\tau^2 \tag{A3f}$$

$$\beta_7 = \frac{\beta_1\beta_2}{\beta_6} \tag{A3g}$$

$$\overline{W}_1 = \frac{d\tau^2}{2}\left(1-\frac{\beta_3}{3}\right) \tag{A4a}$$

$$\dot{W}_1 = d\tau(1-\beta_3) \tag{A4b}$$

$$\eta = \frac{p_0}{p_c} \tag{A4c}$$

$$\xi_0 = \beta \tan\phi \tag{A4d}$$

$$\tan^2\phi = 3-2\xi_0 \tag{A4e}$$

$$\beta = \frac{B}{L} \tag{A4f}$$

and d, h and p_c are selected from Table 1 for the appropriate case, while $\bar{d}$ is equal to the corresponding parameter d with $\eta = 0$.

TABLE 1

	Simply supported beam	Simply supported rectangular plate	Clamped beam	Clamped rectangular plate
d	$\frac{3p_c(\eta-1)}{2\mu}$	$\frac{p_c(\eta-1)(3-\xi_0)}{\mu(2-\xi_0)}$	$\frac{3p_c(\eta-1)}{2\mu}$	$\frac{p_c(\eta-1)(3-\xi_0)}{\mu(2-\xi_0)}$
h	$\frac{6p_c}{\mu H^2}$	$\frac{4p_c\{3-\xi_0+2(1-\xi_0)(3-2\xi_0)\}}{3\mu H^2(2-\xi_0)}$	$\frac{3p_c}{2\mu H^2}$	$\frac{p_c\{3-\xi_0+2(1-\xi_0)(3-2\xi_0)\}}{3\mu H^2(2-\xi_0)}$
p_c	$\frac{2M_0}{L^2}$	$\frac{6M_0}{B^2(3-2\xi_0)}$	$\frac{4M_0}{L^2}$	$\frac{12M_0}{B^2(3-2\xi_0)}$

(b) *Simply supported beams and plates with* $W_m/H \geq \frac{1}{2}$ *and fully clamped beams and plates with* $W_m/H \geq 1$

The results obtained during the first two stages of motion for these particular cases may be expressed in a single form, while the behavior during the third stages must be considered separately. Thus, the velocity $\dot{W}_2$ at the end of the second stage of motion is

$$\frac{\dot{W}_2}{\bar{d}\tau} = \beta_2+(1-\beta_4)\rho_2-\beta_2\beta_5\rho_2^2-(\beta_5+\beta_1\beta_2^2)\frac{\rho_2^3}{3}-\frac{\beta_1\beta_2\rho_2^4}{4}-\frac{\beta_1\rho_2^5}{20} \tag{A5}$$

where the non-dimensionalised duration of the second stage (ρ_2) is the first root for which $\rho_2 \geq 0$ of the polynomial equation

$$X = \frac{\beta_5}{\beta_6}+\beta_7\rho_2+\frac{(\beta_1-\beta_5^2)\rho_2^2}{2\beta_6}-\frac{\beta_5\beta_7\rho_2^3}{3}-\left(\frac{\beta_1\beta_5}{\beta_6}+\beta_6\beta_7^2\right)\frac{\rho_2^4}{12}-\frac{\beta_1\beta_7\rho_2^5}{20}-\frac{\beta_1^2\rho_2^6}{120\beta_6} \tag{A6}$$

$X = \frac{1}{2}$ is used for simply supported beams and rectangular plates, while $X = 1$ corresponds to fully clamped beams and rectangular plates. The constants $\beta_i (i = 1, \ldots, 7)$, $\bar{W}_1, \dot{W}_1, \eta, \xi_0, \phi, d, h, p_c$ and $\bar{d}$ are defined in part (a) of the Appendix.

It may be shown that the permanent transverse deflection is

$$\frac{W_f}{H} = \frac{1}{2}\left\{1+\frac{\beta_6}{4}\left(\frac{\dot{W}_2}{\bar{d}\tau}\right)^2\right\}^{\frac{1}{2}}\left(1-\frac{x}{L}\right) \tag{A7}$$

for a beam with simply supported edges, and

$$\frac{W_f}{H} = \left\{1+\frac{\beta_6}{2}\left(\frac{\dot{W}_2}{\bar{d}\tau}\right)^2\right\}^{\frac{1}{2}}\left(1-\frac{x}{L}\right)$$

for a beam with clamped edges. The maximum permanent transverse deflections are given by the roots of the transcendental equations

$$\beta_8\left(\frac{1}{4}-\frac{W_m^2}{H^2}\right)-\beta_9\log_e\left(\frac{2W_m}{H}\right)+\left(\frac{\beta_1\dot{W}_2}{\beta_6\bar{d}\tau}\right)^2 = 0 \tag{A9}$$

and

$$\frac{\beta_8}{2}\left(1-\frac{W_m^2}{H^2}\right)-2\beta_9\log_e\left(\frac{W_m}{H}\right)+\left(\frac{\beta_1\dot{W}_2}{\beta_6\bar{d}\tau}\right)^2 = 0 \tag{A10}$$

for rectangular plates with simply supported and fully clamped edges, respectively, where

$$\beta_8 = \frac{4p_c\tau^2}{\mu H}\left(1-\xi_0+\frac{1}{2-\xi_0}\right) \tag{A11}$$

and

$$\beta_9 = \frac{2\xi_0 p_c\tau^2}{3\mu H} \tag{A12}$$

and p_c is given in Table 1.

It may be shown that the various predictions for the rectangular plates given by equations (A1), (A9) and (A10) reduce to the corresponding results for beams of length $2B$ when $\beta \to 0$. For convenience of presentation, the results in Fig. 7 for the case $\beta = 0$ correspond to beams of total length $2L$ between the supports.

APPENDIX B

The permanent transverse deflections and response times of simply supported or fully clamped beams and rectangular plates subjected to uniformly distributed pressure pulses are presented in this section. The beams and plates are made from a rigid, perfectly plastic material which yields according to the square yield condition illustrated in Fig. 5.

(a) *Beams*

It may be shown, when using the dissipation relation (23), that the permanent transverse deflection is given by equation (24) when a beam is simply supported, and by

$$\frac{W_f}{H} = \tfrac{1}{2}[\{1+2\eta(\eta-1)[1-\cos(\gamma_1\tau)]\}^{\frac{1}{2}}-1]\left(1-\frac{x}{L}\right) \tag{B1}$$

if a beam is clamped, where

$$\gamma_1^2 = \frac{12M_0}{\mu H L^2}. \tag{B2}$$

The duration of response of both simply supported and fully clamped beams is

$$T = \frac{1}{\gamma_1}\tan^{-1}\left\{\frac{\eta\sin(\gamma_1\tau)}{1-\eta+\eta\cos(\gamma_1\tau)}\right\} \tag{B3}$$

where η is defined by equation (17d) and the appropriate value of the collapse pressure p_c is given in Table 1.

If the dynamic pressure pulse is considered to be impulsive ($p_0\tau = \mu V_0$) then the permanent transverse deflection of a simply supported beam is given by equation (25a) and is

$$\frac{W_f}{H} = \frac{1}{2}\left\{\left(1+\frac{3\lambda}{4}\right)^{\frac{1}{2}}-1\right\}\left(1-\frac{x}{L}\right) \tag{B4}$$

for the corresponding fully clamped case. The duration of response is

$$\frac{1}{\gamma_1}\tan^{-1}(3\lambda)^{\frac{1}{2}} \quad \text{and} \quad \frac{1}{\gamma_1}\tan^{-1}\left(\frac{3\lambda}{4}\right)^{\frac{1}{2}}$$

for the simply supported and clamped cases, respectively. The non-dimensionalised impulse parameter λ is defined by equation (25b).

(b) *Plates*

The maximum transverse deflection of a simply supported rectangular plate is

$$\frac{W_m}{H} = \frac{(3-\xi_0)[\{1+2\eta(\eta-1)(1-\cos\gamma\tau)\}^{\frac{1}{2}}-1]}{4\{1+(1-\xi_0)(2-\xi_0)\}} \tag{B5}$$

where η, ξ_0 and γ are defined by equations (28d), (28e) and (33), respectively, and p_c is given in Table 1. Equation (32) gives the maximum permanent deflection of a fully clamped rectangular plate, while equation (31) gives the duration of response for both simply supported and fully clamped rectangular plates provided the appropriate parameters are used.

If the duration of the dynamic pressure pulse (τ) is sufficiently short so that the loading might be considered as impulsive (i.e. $p_0\tau = \mu V_0$, where V_0 is the initial plate velocity), then the maximum permanent deflection for a simply supported rectangular plate is

$$\frac{W_m}{H} = \frac{(3-\xi_0)[(1+4\Gamma)^{\frac{1}{2}}-1]}{4\{1+(\xi_0-2)(\xi_0-1)\}} \tag{B6}$$

while equation (34) gives the corresponding result for a fully clamped plate. The duration of the motion is

$$T = \frac{1}{\gamma}\tan^{-1}(2\Gamma^{\frac{1}{2}}) \tag{B7}$$

for a simply supported rectangular plate

and

$$T = \frac{1}{\gamma}\tan^{-1}(\Gamma^{\frac{1}{2}}) \tag{B8}$$

for the fully clamped case, where Γ is defined by equation (35).

(*Received* 26 *June* 1970; *revised* 16 *November* 1970)

Абстракт—Описывается приближенный теоретический процесс, с целью оценки остаточных поперечных прогибов балок и пластинок произвольной формы, которые подвергаются действию больмих динамических нагрузок. В анализе учитывается влияние конечных прогибов или изменений геометрии, пока как упругими эффектами пренебрегаются. Иcследуется подобно частный случай полно защемленной прямоугольной пластинки, подверженной действию равномерно приложенного импульса динамического давления. Наблюдается умеренная сходимость между теоретическими предсказаниями и экспериментальными результатами для балок ($\beta = 0$) и прямоугольных пластинок ($\beta = 0{,}593$), изготовленных из материала нечувствительного к скорости деформации.